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EXISTENCE RESULTS, INEQUALITIES AND A PRIORI BOUNDS TO FRACTIONAL BOUNDARY VALUE PROBLEMS

NICHOLAS FEWSTER-YOUNG

Abstract. This paper presents new results to fractional boundary value problems in relation to the existence of solutions and providing *a priori* bounds on all possible solutions subject to a differential inequality. This provides the scope of these results to be applicable to a vast range of problems that may have unrestricted growth to prove the existence of solutions. This contributes to the practically of these results through their application in modelling real world phenomena.

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