

USING MITTAG-LEFFLER FUNCTIONS TO IMPROVE SUFFICIENT CONDITIONS FOR THE UNIQUENESS OF SOLUTIONS TO NABLA BOUNDARY VALUE PROBLEMS

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Abstract. In this paper, the uniqueness of solutions for two prominent classes to nabla fractional boundary value problems are investigated. First, the associated Green's functions are constructed with their equivalent representations and inherent properties are proven. Secondly, the application of the Banach fixed point theorem with sufficient conditions is used to establish the uniqueness and existence of solutions to the considered problems on well-defined spaces with respect to weighted supremum norms. To illustrate the merit, novelty, and applicability of the established results, two examples are presented.

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