

EXISTENCE AND UNIQUENESS OF SOLUTIONS TO TERMINAL VALUE PROBLEMS FOR FRACTIONAL-ORDER DIFFERENTIAL EQUATIONS WITH ADVANCED ARGUMENTS

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Abstract. Our aim is to define the Dzhrbashyan Nersesyan Liouville fractional derivative, elucidate its properties, and apply Banach's fixed point theorem to prove the existence and uniqueness of solutions to terminal value problems for fractional differential equations with advanced arguments involving the Dzhrbashyan Nersesyan Liouville fractional derivative. We provide some examples to showcase the practical application of our results.

Mathematics subject classification (2020): 34A08, 34K37, 34B40.

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