

QUALITATIVE ANALYSIS OF FRACTIONAL DIFFERENTIAL EQUATIONS WITH PROPORTIONAL DELAY

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Abstract. This paper investigates the fractional differential equation (FDE) with proportional delay, a specific form of the fractional order time-dependent delay differential equation (FDDE). We use the Daftardar-Gejji and Jafari Method (DJM) to solve nonlinear FDEs with proportional delay involving the Caputo fractional derivative. We prove existence and uniqueness theorems for these equations and derive convergence results based on the Lipschitz condition. Moreover, we show that DJM solutions are continuously dependent on both the initial conditions and the fractional order. Finally, we derive and prove the convergence of power series solutions for the fractional order pantograph and Ambartsumian equations.

Mathematics subject classification (2020): 26A33, 34A08, 34K06, 34K20.

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