

(ω, c) -ASYMPTOTICALLY PERIODIC MILD SOLUTIONS TO SOME ψ -HILFER FRACTIONAL EVOLUTION EQUATIONS

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Abstract. In this paper, we consider the following abstract ψ -Hilfer fractional differential equation

$$D_{0+}^{\alpha, \beta; \psi} u(t) = Au(t) + f(t, u(t)), \quad 0 < \alpha < 1, \quad 0 < \beta < 1; \quad t \geq 0 \quad (1)$$

where A is the infinitesimal generator of a strongly continuous semigroup $\{T(\theta)\}_{\theta \geq 0}$ on a Banach space \mathbb{X} such that there exist positive constants $M, \lambda \geq 0$ with

$$\|T(\theta)\|_{\mathbb{X}} \leq Me^{-\lambda\theta}, \quad \theta \geq 0. \quad (2)$$

We use the Banach fixed point principle to prove the existence and uniqueness of (ω, c) -asymptotically periodic mild solutions to equation (1). Further, Ulam-Hyers stability results are established.

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