

EXISTENCE RESULTS FOR PANTOGRAPH DIFFERENTIAL EQUATIONS WITH HADAMARD FUNCTIONAL FRACTIONAL DERIVATIVE

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Abstract. This study examines the existence of solutions for various types of fractional pantograph differential equations by employing the Hadamard functional fractional derivative. The results are derived by means of the Banach and Krasnoselskii fixed point theorems. Theoretical results are validated through illustrative examples.

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