

## MOMENTS OF A $q$ -BASKAKOV-BETA OPERATORS IN CASE $0 < q < 1$

A. R. GAIROLA, P. N. AGRAWAL, G. DOBHAL AND K. K. SINGH

**Abstract.** In this paper we obtain the estimates of the central moments for the recently defined  $q$ -analogue of Baskakaov-beta operators. We obtain the evaluation for the rate of convergence in term of the first modulus of smoothness and Voronovskaja-type theorem for these operators.

**Mathematics subject classification (2010):** 41A25, 41A30.

**Keywords and phrases:** Baskakov-beta operators,  $q$ -integer, modulus of continuity.

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