

## SOME SUFFICIENT CONDITIONS FOR THE UNIVALENCE OF AN INTEGRAL OPERATOR

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**Abstract.** Making use of the method of subordination chains, we obtain some sufficient conditions for the univalence of an integral operator. In particular, as special cases, our results imply certain known univalence criteria. A refinement to a quasiconformal extension criterion of the main result, is also obtained.

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