

## DOUBLE SUBORDINATION PRESERVING PROPERTIES FOR GENERALIZED FRACTIONAL DIFFER-INTEGRAL OPERATOR

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**Abstract.** We obtain subordination and superordination preserving properties for the Saigo type generalized fractional differ-integral operator, defined for multivalent functions in the open unit disk. A differential sandwich-type theorem for these multivalent function, and some consequences are also presented.

**Mathematics subject classification (2010):** Primary: 30C80; Secondary: 30C45.

**Keywords and phrases:** Differential subordination and superordination, analytic functions, univalent functions, starlike functions, convex functions, generalized fractional differ-integral operator.

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