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COMPLETE (p,q)–ELLIPTIC INTEGRALS WITH APPLICATION TO A FAMILY OF MEANS

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Abstract. The complete elliptic integrals are generalized by using the generalized trigonometric functions with two parameters. As an application of the integrals, an alternative proof of a result for a family of means by Bhatia and Li, which involves the logarithmic mean and the arithmetic-geometric mean, is given. Moreover, it is shown that a particular relation holds for the generalized integrals.

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