

UPPER BOUNDS FOR A GENERAL LINEAR FUNCTIONAL WITH APPLICATION TO ORTHOGONAL POLYNOMIAL EXPANSIONS

A. MCD. MERCER AND PETER R. MERCER

Abstract. An upper bound on a linear functional satisfying several constraints is found, then used to provide a short and simple proof of convergence, for orthogonal polynomial expansions.

Mathematics subject classification (2010): 42C10, 26D20.

Keywords and phrases: Orthogonal polynomials.

REFERENCES

- [1] HOBSON, E. W., *On a general convergence theorem and the theory of the representation of a function by series of normal functions*, Proc. London Math. Soc. S2-6 (1908), 349–395.
- [2] HOBSON, E. W., *On the representation of a function by series of Legendre functions*, Proc. London Math. Soc. S2-7 (1909), 24–39.
- [3] KHANDEKAR, P. R., *On the bounds for Gegenbauer polynomials*, Amer. Math. Monthly 71 (1964), 1018–1021.
- [4] MERCER, A. MCD. AND MERCER, P. R., *On the sign of $L(f)$ and its error term when f is n -convex*, Demonstratio Mathematica 44 (2011), 223–2312.
- [5] SZEGÖ, G., *Orthogonal Polynomials*, Amer. Math. Soc. Colloquium Publications vol. 23, 1939.
- [6] TITCHMARSH, E. C., *Eigenfunction Expansions Part I, 2nd ed.*, Oxford Univ. Press, 1962.
- [7] WHITTAKER, E. T. AND WATSON, G. N., *A Course of Modern Analysis, 4th ed.*, Cambridge Univ. Press, 1952.