

PROPERTIES FOR THE CLASS OF UNIVALENT FUNCTIONS DEFINED BY DOUBLE SUBORDINATION ASSOCIATED WITH PETAL SHAPED DOMAIN

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Abstract. In this paper, with the help of double subordination the authors introduce a new subclass of analytic univalent functions \mathcal{C}_q^* defined in the open unit disk $\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}$ and examine the initial bounds for the first four coefficients of functions that belong to this newly defined class. Furthermore, we investigate the upper bounds of Fekete-Szegő inequality, Hankel determinant of different orders, Zalcman conjecture, logarithmic coefficients and Inverse coefficients for such family. Our results are new as per the existing literature.

Mathematics subject classification (2020): 30C45, 30C50, 30C55.

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REFERENCES

- [1] R. M. ALI, *Coefficients of the inverse of strongly starlike functions*, Bull. Malays. Math. Sci. Soc. **26** (2003), 63–71.
- [2] M. F. ALI AND A. VASUDEVARAO, *On logarithmic coefficients of some close-to-convex functions*, Proc. Amer. Math. Soc. **146** (2018), 1131–1142.
- [3] M. ARIF, M. RAZA, H. TANG, S. HUSSAIN AND H. KHAN, *Hankel determinant of order three for familiar subclass of analytic functions related with sine function*, Open Math. **17** (2019), 1615–1630.
- [4] K. O. BABALOLA, *On $H_3(1)$ Hankel determinant for some classes of univalent functions*, Inequal. Theory Appl. **6** (2007), 1–7.
- [5] D. BANSAL AND J. SOKOL, *Zalcman conjecture for some subclass of analytic functions*, J. Fract. Cal. Appl. **8** (1) (2017), 1–5.
- [6] D. BREAZ, T. PANIGRAHI, S. M. EL-DEEB, E. PATTNAYAK AND S. SIVASUBRAMANIAN, *Coefficient bounds for two subclasses of analytic functions involving a limaçon shaped domain*, Symmetry **16** (2024), 183, <https://doi.org/10.3390/sym16020183>.
- [7] C. CARATHÉODORY, *Über den Variabilitätsbereich der Koeffizienten von Potenzreihen, die gegebene Werte nicht annehmen*, Math. Ann. **64** (1) (1907), 95–115.
- [8] N. E. CHO, B. KOWALCZYK, O. S. KWON, A. LECKO AND Y. J. SIM, *On the third logarithmic coefficient in some subclasses of close-to-convex functions*, Rev. Real. Acad. Cienc. Exactas Fis. Nat. Ser. A Mat. **114** (2020), 1–14.
- [9] P. L. DUREN, *Univalent Functions, Grundlehren der mathematischen Wissenschaften*, vol. 259, Springer New York, NY, 2001.
- [10] M. G. KHAN, B. AHMAD, G. MURUGUSUNDARAMOORTHY, W. K. MASHWANI, S. YALCIN, T. G. SHABA AND Z. SALLEH, *Third Hankel determinant and Zalcman functional for a class of starlike functions with respect to symmetric points related with sine function*, J. Math. Comput. Sci. **25** (2022), 29–36.
- [11] B. KOWALCZYK, A. LECKO AND Y. J. SIM, *The sharp bound of the Hankl determinant of the third kind for convex functions*, Bull. Aust. Math. Soc. **97** (2018), 435–445.
- [12] U. P. KUMAR AND A. VASUDEVARAO, *Logarithmic coefficients for certain subclasses of close-to-convex functions*, Monats. Math. **187** (2018), 543–563.

- [13] O. S. KWON, A. LECKO AND Y. J. SIM, *The bound of the Hankel determinant of the third kind for starlike functions*, Bull. Malays. Math. Sci. Soc. **42** (2019), 767–780.
- [14] A. LECKO, Y. J. SIM AND B. SMIAROWSKA, *The sharp bound of the Hankel determinant of the third kind for starlike functions of order $\frac{1}{2}$* , Complex Anal. Oper. Theory **13** (2019), 2231–2238.
- [15] W. C. MA, *The Zalcman conjecture for close-to-convex functions*, Proc. Amer. Math. Soc. **104** (3) (1988), 741–744.
- [16] W. C. MA AND D. A. MINDA, *A unified treatment of some special classes of univalent functions*, in: Proceedings of the Conference on Complex Analysis (Tianjin, 1992); Int. Press: Cambridge, MA, USA, 1994, 157–169.
- [17] K. MARIMUTHU, J. UMA AND T. BULBOACĂ, *Coefficient estimates for starlike and convex functions associated with cosine function*, Hacet. J. Math. Stat. **52** (3) (2023), 596–618.
- [18] W. NOONAN AND D. K. THOMAS, *On the second Hankel determinant of areally mean p -valent function*, Trans. Amer. Math. Soc. **223** (1976), 337–346.
- [19] T. PANIGRAHI, E. PATTNAYAK AND R. M. EL-ASHWAH, *Estimate on logarithmic coefficients of Kamali-type starlike functions associated with four-leaf shaped domain*, Surveys Math. Appl. **19** (2024), 41–55.
- [20] C. POMMERENKE, *Univalent Function*, Vandenhoeck and Ruprecht: Göttingen, Germany, 1975.
- [21] R. K. RAINA AND J. SOKÓŁ, *Some properties related to a certain class of starlike functions*, C. R. Math. Acad. Sc. Paris **353** (11) (2015), 973–978,
<https://doi.org/10.1016/j.crma.2015.09.011>.
- [22] H. M. SRIVASTAVA, B. KHAN, N. KHAN, M. TAHIR, S. AHMAD AND N. KHAN, *Upper bound of the third Hankel determinant for a subclass of q -starlike function associated with the q -exponential function*, Bull. Sci. Math. **167** (2021), Art. no. 102942, 16 pages.
- [23] D. K. THOMAS, *On logarithmic coefficients of close-to-convex functions*, Proc. Amer. Math. Soc. **144** (2016), 1681–1687.
- [24] P. ZAPRAWA, *Third Hankel determinant for subclasses of univalent functions*, Mediterr. J. Math. **14** (2017), 14–19.
- [25] P. ZAPRAWA, *Initial logarithmic coefficients for functions starlike with respect to symmetric points*, Bol. Soc. Mat. Max. **27** (2021), 1–13.
- [26] P. ZAPRAWA, M. OBRADOVIĆ, AND N. TUNESKI, *Third Hankel determinant for univalent starlike functions*, Rev. Real Acad. Cienc. Exactas Fis. Nat. Ser. A-Mat. **49** (2021).