

EXPLORING GENERALIZED IDEAL CONVERGENCE WITH ORLICZ FUNCTIONS IN NEUTROSOPHIC n -NORMED SPACES

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Abstract. This paper introduces the concept of \mathcal{I}_λ -convergence within the framework of neutrosophic n -normed spaces. Leveraging this notion alongside Orlicz functions, we construct novel sequence spaces that extend the theoretical landscape of neutrosophic n -normed linear spaces. Our investigation delves into the structural properties of these spaces, exploring fundamental attributes such as linearity and Hausdorffness while establishing significant results. This study deepens the understanding of neutrosophic n -normed spaces and also contributes to the broader mathematical framework by unveiling new perspectives in sequence space theory.

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