

ON THE CONVERGENT AND NULL GENERALIZED MOTZKIN SEQUENCE SPACES

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Abstract. The BK sequence spaces $c(\mathcal{G})$ and $c_0(\mathcal{G})$ generated by generalized Motzkin (GM) matrix \mathcal{G} are constructing in this study. The Schauder bases of these spaces are obtained and the inclusion relations are presented. Additionally, the α -, β - and γ -duals of $c(\mathcal{G})$ and $c_0(\mathcal{G})$ are determined and finally, the necessary and sufficient conditions for a matrix to be in the class of matrices containing new spaces are explored.

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REFERENCES

- [1] M. AIGNER, *Motzkin numbers*, Europ. J. Combinatorics, **19**, (1998), 663–675.
- [2] B. ALTAY AND F. BAŞAR, *Certain topological properties and duals of the matrix domain of a triangle matrix in a sequence space*, J. Math. Anal. Appl., **336**, (2007), 632–645.
- [3] F. BAŞAR, *Summability Theory and Its Applications*, CRC Press/Taylor & Francis Group, 2nd ed., Boca Raton · London · New York, 2022.
- [4] J. BOOS, *Classical and Modern Methods in Summability*, Oxford Science Publications, Oxford University Press, 2000.
- [5] M. C. DAĞLI, *A novel conservative matrix arising from Schröder numbers and its applications*, Linear Multilinear Algebra, **71**, 8 1338–1351.
- [6] M. C. DAĞLI, *Matrix mappings and compact operators for Schröder sequence spaces*, Turkish J. Math., **46**, (2022), 2304–2320.
- [7] S. DEMİRİZ AND S. ERDEM, *Mersenne matrix operator and its application in p -summable sequence space*, Communications in Advanced Mathematical Sciences, **7**, 1 (2024), 42–55.
- [8] S. DEMİRİZ, A. ŞAHİN AND S. ERDEM, *Some topological and geometric properties of novel generalized Motzkin sequence spaces*, Rend. Circ. Mat. Palermo, II. Ser., **74**, 136 (2025).
- [9] R. DONAGHEY AND L. W. SHAPIRO, *Motzkin numbers*, J. Combin. Theory Ser. A, **23**, (1977). 291–301.
- [10] R. DONAGHEY, *Restricted plane tree representations for four Motzkin-Catalan equations*, J. Combin. Theory Ser. B, **22**, (1977), 114–121.
- [11] S. ERDEM AND S. DEMİRİZ, *On the new generalized block difference sequence space*, Appl. Appl. Math. (AAM), Special Issue **5**, (2019), 68–83.
- [12] S. ERDEM, S. DEMİRİZ AND A. ŞAHİN, *Motzkin sequence spaces and Motzkin core*, Numer. Funct. Anal. Optim., **45**, 4 (2024), 1–21.
- [13] S. ERDEM, *Compact operators on the Motzkin sequence space $c_0(\mathcal{M})$* , Journal of New Results in Science, **13**, 2 (2024), 109–118, <https://doi.org/10.54187/jnrs.1517251>.
- [14] S. ERDEM, *Compact operators on the new Motzkin sequence spaces*, AIMS Mathematics, **9**, 9 (2024), 24193–24212.
- [15] F. HARARY AND R. READ, *The enumeration of tree-like polyhexes*, Proc. Edinburgh Math. Soc., **17**, (1970), 1–13.
- [16] M. İLKHAN, P. Z. ALP AND E. E. KARA, *On the spaces of linear operators acting between asymmetric cone normed spaces*, Mediterr. J. Math., **15**, 136 (2018).

- [17] M. İLKHAN, *A new conservative matrix derived by Catalan numbers and its matrix domain in the spaces c and c_0* , Linear Multilinear Algebra, **68**, 2 (2019), 417–434.
- [18] M. İLKHAN AND E. E. KARA, *Matrix transformations and compact operators on Catalan sequence spaces*, J. Math. Anal. Appl., **498**, (2021), 124925.
- [19] E. E. KARA, M. BAŞARIR, *An application of Fibonacci numbers into infinite Toeplitz matrices*, Caspian Journal of Mathematical Sciences, **1**, 1 (2012), 43–47.
- [20] E. E. KARA, *Some topological and geometrical properties of new Banach sequence spaces*, J. Inequal. Appl., **2013**, 38 (2013).
- [21] M. KARAKAŞ AND M. C. DAĞLI, *Some topologic and geometric properties of new Catalan sequence spaces*, Advances in Operator Theory, **8**, 14 (2023), 15 pages.
- [22] M. KARAKAŞ AND H. KARABUDAK, *An application on the Lucas numbers and infinite Toeplitz matrices*, Cumhuriyet Sci. J., **38**, 3 (2017), 557–562.
- [23] M. KARAKAŞ AND A. M. KARAKAŞ, *New Banach sequence spaces that is defined by the aid of Lucas numbers*, Iğdır Univ. J. Inst. Sci. Tech., **7**, 4 (2017), 103–111.
- [24] M. KARAKAŞ, *On the sequence spaces involving Bell numbers*, Linear Multilinear Algebra, **71**, 14 (2022), 2298–2309.
- [25] T. MOTZKIN, *Relations between hypersurface cross ratios, and a combinatorial formula for partitions of a polygon, for permanent preponderance, and for nonassociative products*, Bull. Amer. Math. Soc., **54**, (1948), 352–360.
- [26] M. MURSALEEN AND F. BAŞAR, *Sequence spaces: Topic in Modern Summability Theory*, CRC Press, Taylor Francis Group, Series: Mathematics and its applications, Boca Raton · London · New York, 2020.
- [27] OEIS FOUNDATION INC., *The On-Line Encyclopedia of Integer Sequences*, Retrieved Sep. 19, 2024, from <http://oeis.org>.
- [28] M. STIEGLITZ AND H. TIETZ, *Matrix transformationen von folgenraumen eine ergebnisbersicht*, Math Z. 1977; 154: 1–16.
- [29] Z.-W. SUN, *Congruences involving generalized central trinomial coefficients*, Sci. China Math., **57**, (2014), 1375–1400.
- [30] Y. WANG AND Z.-H. ZHANG, *Combinatorics of Generalized Motzkin Numbers*, Journal of Integer Sequences, **18**, (2015), Article 15.2.4.
- [31] T. YAYING, B. HAZARIKA AND S. A. MOHIUDDINE, *Domain of Padovan q -difference matrix in sequence spaces ℓ_p and ℓ_∞* , Filomat, **36**, 3 (2022), 905–919.
- [32] T. YAYING, B. HAZARIKA, O. M. KALTHUM S. K. MOHAMED AND AWAD A. BAKERY, *On new Banach sequence spaces involving Leonardo numbers and the associated mapping ideal*, Journal of Function Spaces, **2022**, 8269000 (2022), 21 pages.
- [33] A. WILANSKY, *Summability Through Functional Analysis*, Amsterdam-New York-Oxford: North-Holland Mathematics Studies **85**, 1984.