

## GENERAL ONE-SIDED MAXIMAL OPERATORS ON WEIGHTED ONE-SIDED HERZ SPACES

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**Abstract.** This paper establishes the mapping properties of the general one-sided maximal operators from the weak weighted one-sided Herz spaces to the weighted one-sided Herz spaces. These mapping properties are inherited from the corresponding mapping properties from the Lebesgue spaces.

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