

QUATERNARY ELLIPTIC FUNCTIONS IN THE SPIRIT OF DU VAL

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Abstract. In 1964 Patrick Du Val introduced and studied his ‘ternary’ elliptic functions as cube roots of the suitably modified first derivative p' of an arbitrary Weierstrass \wp -function. We introduce and study ‘quaternary’ elliptic functions as fourth roots of its suitably modified second derivative p'' .

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