

ON SOME CRITERIA FOR STARLIKENESS OF RATIONAL FUNCTIONS

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Abstract. In the present paper, we investigate a new class of analytic functions $\mathcal{V}_{\alpha,\mu}(A,B)$ and subclasses of it, which consists of analytic functions f where satisfies the condition

$$\mathcal{V}_{\alpha,\mu}(A,B) = \left\{ f \in \mathcal{A} : (1-\alpha) \left(\frac{z}{f(z)} \right)^\mu + \alpha \left(\frac{z}{f(z)} \right)^{\mu+1} f'(z) \prec \frac{1+Az}{1+Bz} \right\}.$$

By making use of differential subordination, we find conditions on the parameters of α, μ, A and B which guarantee univalence or starlikeness of the members of that class. Our results will generalize or improve the earlier results obtained by other researches. Also we provide some new criteria for rational functions to be univalent or starlike.

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