

## A FOUR-DIMENSIONAL TAUBERIAN THEOREM FOR BLOCK-STRETCHED SEQUENCES

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**Abstract.** This paper presents a Tauberian theorem for four-dimensional RH-regular summability methods applied to double sequences and their block stretchings. The work extends Dawson's lemma and Maddox's theorem from single sequence theory to the double sequence setting, establishing necessary conditions for matrices that sum every block stretching. The main result characterizes block-finite matrices and provides a complete Tauberian-type theorem for block-stretched double sequences.

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