

SOME REMARKS ON OPEN PROBLEMS FOR NIELSEN'S BETA FUNCTION

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Abstract. Using the convolution properties of the Laplace transforms, the generalized Schwartz inequality, and the monotonicity of the Nielsen beta function, we address the open problems related to the Nielsen's beta function. Consequently, we improve the validity of the existing inequalities.

Mathematics subject classification (2020): 33B15, 33Bxx, 26D07, 26A48.

Keywords and phrases: Nielsen's beta function, open problems, inequalities.

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