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SOME REMARKS ON OPEN PROBLEMS FOR NIELSEN'S BETA FUNCTION

K. JYOTHI, B. RAVI* AND A. VENKATA LAKSHMI

Abstract. Using the convolution properties of the Laplace transforms, the generalized Schwartz inequality, and the monotonicity of the Nielsen beta function, we address the open problems related to the Nielsen's beta function. Consequently, we improve the validity of the existing inequalities.

1. Introduction

It is well known that the Euler gamma function $\Gamma(\nu)$ is defined for $\nu > 0$ as $\Gamma(\nu) = \int_0^\infty e^{-t} t^{\nu-1} dt$. The function $\psi(\nu) = \frac{\Gamma'(\nu)}{\Gamma(\nu)}$, the logarithmic derivative of the gamma function, is called psi or digamma function, and $\psi^{(n)}(\nu)$ for $n \in \mathbb{N}$ is called the polygamma functions.

The Nielsen beta function $\beta(\nu)$ is defined for $\nu > 0$ as

$$\beta(\nu) = \frac{1}{2}\left\{\psi\left(\frac{\nu+1}{2}\right) - \psi\left(\frac{\nu}{2}\right)\right\}. \tag{1}$$

and the integral representation of $\beta(\nu)$ is

$$\beta(\nu) = \int_0^\infty \frac{e^{-\nu t}}{1+e^{-t}} dt. \tag{2}$$

By successive differentiation of (1) and (2), we obtained

$$\beta^{(n)}(\nu) = \frac{1}{2^{n+1}}\left\{\psi^{(n)}\left(\frac{\nu+1}{2}\right) - \psi^{(n)}\left(\frac{\nu}{2}\right)\right\}, \tag{3}$$

and

$$\beta^{(n)}(\nu) = (-1)^n \int_0^\infty \frac{t^n e^{-\nu t}}{1+e^{-t}} dt. \tag{4}$$

and the series representation is

$$(-1)^n \beta^{(n)}(\nu) = n! \sum_{k=0}^{\infty} \frac{(-1)^k}{(k+\nu)^{n+1}}. \tag{5}$$

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* Corresponding author.

For additional properties and inequalities on the function refer to [1, 2, 5–7, 9, 16, 17] and [3, 11, 15]. The generalized Schwartz inequality [4],

$$\int_a^b g(t)\,[f(t)]^m\,dt \cdot \int_a^b g(t)\,[f(t)]^n\,dt \geqslant \left[\int_a^b g(t)\,[f(t)]^{\frac{m+n}{2}}\,dt\right]^2, \tag{6}$$

where f and g are two nonnegative functions of real variables and m and n belong to a set $\mathscr{S} \subseteq \mathbb{R}$, such that the integrals in (6) exist.

A function f is said to be completely monotonic on the interval (a,b), where $-\infty \leqslant a < b \leqslant \infty$, if f has derivatives of all orders on (a,b) and satisfies the inequality:

$$(-1)^n f^{(n)}(v) \geqslant 0, \quad \text{for all} \quad v \in (a,b) \quad \text{and} \quad n \in \{0\} \cup \mathbb{N}, \tag{7}$$

see [14]. The famous Bernstein's theorem [14, p.161, Theorem 12b]: A function $f(v)$ is completely monotonic on $(0,\infty)$ if and only if

$$f(v) = \int_0^\infty e^{-vt} d\sigma(t), \;\; v \in (0,\infty), \tag{8}$$

where $\sigma(\tau)$ is non-decreasing and the integral in (8) converges for $v \in (0,\infty)$.

In sec. 2, we answer four open problems related to the Nielsen beta functions proposed in [12], stated as

PROBLEM 1. For $v > 0$, show whether the function

$$F(v) = \frac{v\beta^{(n+1)}(v)}{\beta^{(n)}(v)} \tag{9}$$

is increasing or decreasing.

PROBLEM 2. For $v > 0$, show whether the function

$$G(v) = \frac{v\beta^{(n+1)}(v)}{(\beta^{(n)}(v))^2} \tag{10}$$

is increasing or decreasing.

PROBLEM 3. For $v > 0$, show whether the function

$$H(v) = \beta^{(n)}(v)\beta^{(n+2)}(v) - 2\left(\beta^{(n+1)}(v)\right)^2 \tag{11}$$

is positive or negative and improve Theorem 2.6 in [10]. Three of the four open problems have already been addressed in the literature (see [11, 15]). Our present work adopts a different approach to these problems, and improving some previously known results.

2. Answers to open problems

In order to prove the open problems, we required the following lemmas.

LEMMA 1. *For any fixed integer $n \geqslant 2$, let*

$$I_n(a) = \int_0^1 [(2n-1)\nu^2 - 1] f_n(a(1-\nu)) f_n(a(1+\nu))\, d\nu,$$

where

$$f_n(t) = \frac{t^n}{1+e^{-t}}.$$

Then $I_n(a) < 0$ for all $a > 0$.

Proof. Let $h_n(\nu) = [(2n-1)\nu^2 - 1](1-\nu^2)^{n-2}$ and $u(\nu;a) = (1-\nu^2)\vartheta(\nu;a)$ where

$$\vartheta(\nu;a) = \frac{a(1-\nu)}{1+e^{-a(1-\nu)}} \frac{a(1+\nu)}{1+e^{-a(1+\nu)}}.$$

Then

$$I_n(a) = a^{2(n-1)} \int_0^1 h_n(\nu) u(\nu;a) d\nu. \tag{12}$$

Now, we show that $\nu \mapsto \vartheta(\nu;a)$ is strictly decreasing on $(0,1)$. In order to prove that

$$\frac{\partial}{\partial \nu} \vartheta(\nu;a) < 0 \quad 0 < \nu < 1; \;\; a > 0.$$

Define $k(\nu;a) = \log \vartheta(\nu;a)$. Then we have

$$\frac{\partial}{\partial \nu} \vartheta(\nu;a) = a[w(a(1+\nu)) - w(a(1-\nu))],$$

where

$$w(\mu) = \frac{1}{\mu} + \frac{1}{1+e^{\mu}}.$$

Since

$$\frac{d}{d\mu} w(\mu) = -\left(\frac{1}{\mu^2} + \frac{e^{\mu}}{(1+e^{\mu})^2}\right) < 0 \quad \text{for} \quad \mu > 0,$$

and since $0 < a(1-\nu) < a(1+\nu)$ we have

$$w(a(1+\nu)) < w(a(1-\nu)),$$

so that

$$\frac{\partial}{\partial \nu} \vartheta(\nu;a) = \vartheta(\nu;a) \frac{\partial}{\partial \nu} k(\nu;a) < 0,$$

$\vartheta(\nu;a)$ is decreasing and hence $u(\nu;a)$ is decreasing on $(0,1)$, and hence,

$$h_n(\nu) u(\nu;a) < h_n(\nu) u\left(\frac{1}{\sqrt{2n-1}};a\right) \quad \text{for } 0 < \nu < 1, \;\; \nu \neq \frac{1}{\sqrt{2n-1}}, \;\; a > 0. \tag{13}$$

From (12), (13) and

$$\int_0^1 h_n(\nu)\nu = \left[-\nu(1-\nu^2)^{n-1}\right]_0^1 = 0,$$

it follows that

$$I_n(a) < a^{2(n-1)} u\left(\frac{1}{\sqrt{2n-1}}; a\right) \int_0^1 h_n(\nu)d\nu = 0. \quad \square$$

LEMMA 2. (Convolution theorem for the Laplace transforms [14, pp. 91–92]) *Let $f_k(\nu)$ for $k = 1,2$ be piecewise continuous in arbitrary finite intervals in $(0,\infty)$. If there exist some constants τ_k and c_k such that $|f_k(\nu)| \leqslant \tau_k e^{c_k \nu}$ for $k = 1,2,$, then*

$$\int_0^\infty f_1(u)e^{-su}du \int_0^\infty f_2(\nu)e^{-s\nu}d\nu = \int_0^\infty \left[\int_0^t f_1(u)f_2(t-u)du\right] e^{-st}dt. \tag{14}$$

In the next theorem we show that the function

$$F_n(\nu;\alpha) = (\beta^{(n)}(\nu))^2 - \alpha\beta^{(n-1)}(\nu)\beta^{(n+1)}(\nu),$$

$\nu > 0$ is strictly completely monotonic on $(0,\infty)$.

THEOREM 1. *The function $F_n(\nu;\alpha) = (\beta^{(n)}(\nu))^2 - \alpha\beta^{(n-1)}(\nu)\beta^{(n+1)}(\nu),\ \nu > 0$ where, $n \geqslant 2$ is strictly completely monotonic on $(0,\infty)$ if $\alpha \leqslant \frac{n-1}{n}$.*

Proof. By the equation (4) we have

$$(-1)^{n+1}\beta^{(n+1)}(\nu) = \int_0^\infty e^{-\nu t} t f_n(t)dt \tag{15}$$

$$(-1)^{n+2}\beta^{(n+2)}(\nu) = \int_0^\infty e^{-\nu t} t^2 f_n(t)dt \tag{16}$$

where f_n is as in Lemma (1). It follows from (4), (15) and (16) that

$$\begin{aligned} F_n\left(\nu;\frac{n-1}{n}\right) &= \left[(-1)^{n+1}\beta^{(n+1)}(\nu)\right]^2 - \frac{n-1}{n}\left[(-1)^n\beta^{(n)}(\nu)(-1)^{n+2}\beta^{(n+2)}(\nu)\right] \\ &= \left(\int_0^\infty e^{-\nu t} t f_n(\nu)\,dt\right)\left(\int_0^\infty e^{-\nu t} t f_n(\nu)\,dt\right) \\ &\quad - \frac{n-1}{n}\left(\int_0^\infty e^{-\nu t} f_n(\nu)\,dt\right)\left(\int_0^\infty e^{-\nu t} t^2 f_n(\nu)\,dt\right). \end{aligned}$$

By convolution of the Laplace transform (14) we obtain

$$F_n(\nu;\frac{n-1}{n}) = \int_0^\infty e^{-\nu t} g_n(t)dt,$$

where

$$g_n(t) = \int_0^t \left(t - \frac{2n-1}{n}s\right) s f_n(s) f_n(t-s)ds.$$

Let $s = \frac{t}{2}(1+\nu)$. Then

$$g_n(t) = \frac{t^3}{8n}\int_{-1}^{1}[1-(2n-1)\nu-(2n-1)\nu^2]f_n\left(\frac{t}{2}(1-\nu)\right)f_n\left(\frac{t}{2}(1+\nu)\right)d\nu.$$

Since the function $\nu \mapsto \nu f_n\left(\frac{t}{2}(1-\nu)\right)f_n\left(\frac{t}{2}(1+\nu)\right)$ is an odd function, we obtain

$$\begin{aligned} g_n(t) &= \frac{t^3}{8n}\int_{-1}^{1}[1-(2n-1)\nu^2]f_n\left(\frac{t}{2}(1-\nu)\right)f_n\left(\frac{t}{2}(1+\nu)\right)d\nu \\ &= -\frac{t^3}{4n}I_n\left(\frac{t}{2}\right). \end{aligned}$$

where I_n as in lemma (1). Hence, it follows from (1) and (7) that

$$(-1)^m\frac{d^m}{d\nu^m}F_n\left(\nu;\frac{n-1}{n}\right) = \int_0^\infty e^{-\nu t}t^m g_n(t)dt > 0, \quad (\nu > 0,\ m = 1,2,\cdots).$$

Hence the function $F_n\left(\nu;\frac{n-1}{n}\right)$ is strictly completely monotonic on $(0,\infty)$ The series representation (5) implies that $(-1)^n\beta^{(n)}(\nu)$, $(n \geqslant 1)$ is strictly completely monotonic on $(0,\infty)$. Since a positive linear combination of strictly completely monotonic functions is also strictly completely monotonic, we get from

$$F_n(\nu;\alpha) = F_n\left(\nu;\frac{n-1}{n}\right) + \left(\frac{n-1}{n}-\alpha\right)((-1)^n\beta^{(n)}(\nu)(-1)^{n+1}\beta^{(n+1)}(\nu))$$

that the function $F_n(\nu;\alpha)$ is strictly completely monotonic on $(0,\infty)$ if $\alpha \leqslant \frac{n-1}{n}$. □

As a direct consequence of Theorem (1), we obtain the following corollary, which addresses the third open problem and the same conclusion has been given in [15].

COROLLARY 1. *For $n \in \mathbb{N}$, the function*

$$H(\nu) = \beta^{(n)}(\nu)\beta^{(n+2)}(\nu) - 2(\beta^{(n+1)}(\nu))^2$$

is negative on $(0,\infty)$.

In the following theorem, we address the second open problem.

THEOREM 2. *For $\nu > 0$, the function*

$$G(\nu) = \frac{\nu\beta^{(n+1)}(\nu)}{(\beta^{(n)}(\nu))^2} \tag{17}$$

is increasing when n is odd and decreasing when n is even.

Proof. Differentiating equation (17) yields

$$G'(v) = \frac{(\beta^{(n)}(v))^2\left[v(v)\beta^{(n+2)}(v)+\beta^{(n+1)}(v)\right]-2v\beta^{(n)}(v)(\beta^{(n+1)}(v))^2}{(\beta^{(n)}(v))^4}$$
$$= \frac{1}{(\beta^{(n)}(v))^3}\left[v\left(\beta^{(n)}(v)\beta^{(n+2)}(v)-2(\beta^{(n+1)}(v))^2\right)+\beta^{(n)}(v)\beta^{(n+1)}(v)\right].$$

For even values of n, $\beta^{(n)}(v)>0$ and from the Corollary (1) that

$$\beta^{(n)}(v)\beta^{(n+2)}(v)-2(\beta^{(n+1)}(v))^2<0,$$

and $\beta^{(n)}(v)\beta^{(n+1)}(v)<0$. Therefore, $G'(v)<0$, which implies that $G(v)$ is a decreasing function on $(0,\infty)$. For odd values of n, $\beta^{(n)}(v)<0$ and from the Corollary (1) that

$$\beta^{(n)}(v)\beta^{(n+2)}(v)-2(\beta^{(n+1)}(v))^2<0,$$

and $\beta^{(n)}(v)\beta^{(n+1)}(v)<0$. Therefore, $G'(v)>0$, which implies that $G(v)$ is an increasing function on $(0,\infty)$. □

REMARK 1. The proof of this theorem is given in [11] using a different approach.

In the following theorem, we address the first open problem.

THEOREM 3. *For $v>0$, the function*

$$F(v)=\frac{v\beta^{(n+1)}(v)}{\beta^{(n)}(v)}$$

is neither increasing nor decreasing when $n=1$.

Proof. For $n=1$, using equation (3), we have

$$F(v)=\frac{v\beta''(v)}{\beta'(v)}$$
$$=\frac{v}{2}\left(\frac{\psi''\left(\frac{v+1}{2}\right)-\psi''\left(\frac{v}{2}\right)}{\psi'\left(\frac{v+1}{2}\right)-\psi'\left(\frac{v}{2}\right)}\right).$$

Utilizing equation (3), we obtain

$$F(1)=-\frac{18\zeta(3)}{\pi^2},\quad F(2)=\frac{48-36\zeta(3)}{\pi^2-12},\quad F(3)=\frac{63-54\zeta(3)}{\pi^2-9}.$$

Further, we compute

$$F(1)-F(2)=\frac{216\zeta(3)+6\pi^2(3\zeta(3)-8)}{\pi^2(\pi^2-12)}\approx 0.0260557, \tag{18}$$

and

$$F(2)-F(3)=\frac{324-15\pi^2+18(\pi^2-18)\zeta(3)}{(\pi^2-12)(\pi^2-9)}\approx -0.0207099. \tag{19}$$

From equations (18) and (19), we observe that $F(1)>F(2)$ and $F(2)<F(3)$, which implies that $F(\nu)$ is neither increasing nor decreasing when $n=1$. □

THEOREM 4. *For $m,n\in\mathbb{N}$, then*

$$\beta^{(m)}(\nu)\beta^{(n)}(\nu)\geqslant\left(\beta^{(\frac{m+n}{2})}(\nu)\right)^2, \tag{20}$$

where $\frac{m+n}{2}$ is an integer.

Proof. By the equation (4) we have

$$\beta^{(n)}(\nu)=(-1)^n\int_0^\infty\frac{t^n}{1+e^{-t}}e^{-\nu t}dt,\quad \nu>0,\quad n=1,2,\ldots \tag{21}$$

We choose the integers m and n both even or both odd, in such a way that $(m+n)/2$ is an integer. By (6) with $g(t)=\frac{e^{-\nu t}}{1+e^{-t}}$, $f(t)=t$ and $a=0$, $b=\infty$, we get

$$\left(\int_0^\infty\frac{e^{-\nu t}}{1+e^{-t}}t^n dt\right)\cdot\left(\int_0^\infty\frac{e^{-\nu t}}{1+e^{-t}}t^m dt\right)\geqslant\left(\int_0^\infty\frac{e^{-\nu t}}{1+e^{-t}}t^{(m+n)/2}dt\right)^2, \tag{22}$$

that is,

$$\beta^{(m)}(\nu)\beta^{(n)}(\nu)\geqslant\left(\beta^{(m+n)/2}(\nu)\right)^2,\quad m,n=1,3,5,\ldots\quad\text{or}\quad m,n=2,4,6,\ldots. \tag{23}$$

The proof is complete. □

REMARK 2. When $m=n+2$ we find

$$\beta^{(n)}(\nu)\beta^{(n+2)}(\nu)\geqslant\left(\beta^{(n+1)}\right)^2\quad n=1,2,\cdots,\ \ \nu>0. \tag{24}$$

This inequality also improves Theorem 2.6 in [10].

COROLLARY 2. *The function*

$$F_1(\nu)=\frac{\beta^{(n+1)}(\nu)}{\beta^{(n)}(\nu)} \tag{25}$$

is increasing for all $\nu>0$ and $n\in\mathbb{N}$.

Proof. By differentiating the equation (25), and by the remark (24) we obtain

$$F_1'(\nu)=\frac{\beta^{(n)}(\nu)\beta^{(n+2)}(\nu)-(\beta^{(n+1)}(\nu))^2}{(\beta^{(n)}(\nu))^2}\geqslant 0,$$

for all $\nu>0$ and $n\in\mathbb{N}$. This implies that $F_1(\nu)$ is increasing. This completes the proof of the corollary. □

THEOREM 5. *When $n \in \mathbb{N}$ is odd, the function*

$$f(\nu) = \beta^{(n)}(\nu) + \nu\beta^{(n+1)}(\nu), \tag{26}$$

is positive on $(0,\infty)$ and when $n \in \mathbb{N}$ is even the function

$$f(\nu) = \beta^{(n)}(\nu) + \nu\beta^{(n+1)}(\nu), \tag{27}$$

is negative on $(0,\infty)$.

Proof. Using (4) and the convolution theorem for the Laplace transform, we proceed as follows. For $\nu > 0$,

$$\begin{aligned}
\frac{f(\nu)}{\nu} &= \frac{1}{\nu}\beta^{(n)}(\nu) + \beta^{(n+1)}(\nu) \\
&= \int_0^\infty e^{-\nu t}\, dt \int_0^\infty \frac{(-1)^n t^n e^{-\nu t}}{1+e^{-t}}\, dt + \int_0^\infty \frac{(-1)^{n+1} t^{n+1} e^{-\nu t}}{1+e^{-t}}\, dt \\
&= \int_0^\infty (-1)^n \left(\int_0^t \frac{u^n}{1+e^{-u}}\, du - \frac{t^{n+1}}{1+e^{-t}} \right) e^{-\nu t}\, dt \\
&= \int_0^\infty A_n(t) e^{-\nu t}\, dt,
\end{aligned}$$

where

$$A_n(t) = (-1)^n \left[\int_0^t \frac{u^n}{1+e^{-u}}\, du - \frac{t^{n+1}}{1+e^{-t}} \right].$$

Now, we observe that $A_n(0) = \lim_{t\to 0^+} A_n(t) = 0$ and

$$\begin{aligned}
A_n'(t) &= (-1)^n \left[\frac{t^n}{1+e^{-t}} - \frac{(n+1)t^n}{1+e^{-t}} - \frac{t^{n+1}e^{-t}}{(1+e^{-t})^2} \right] \\
&= \frac{(-1)^{n+1} t^n}{1+e^{-t}} \left[n + \frac{te^{-t}}{1+e^{-t}} \right].
\end{aligned}$$

When n is odd, $A_n'(t) > 0$, which implies that $A_n(t)$ is increasing. Consequently, for $t > 0$, $A_n(t) > 0$, implying $\frac{f(\nu)}{\nu} > 0$, and thus $f(\nu) > 0$. When n is even, $A_n'(t) < 0$, implying that $A_n(t)$ is decreasing. Therefore, for $t > 0$, $A_n(t) < 0$, implying $\frac{f(\nu)}{\nu} < 0$, and thus $f(\nu) < 0$. This completes the proof. $\square$

REMARK 3. The proof of this theorem is given in [15] using a different approach.

Conflict of interest. The authors declare that they have no conflict of interest.

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K. Jyothi
Department of Mathematics
Osmania University
Telangana State, 500007, India
e-mail: jyothinaveen2008@gmail.com

B. Ravi
Department of Mathematics
Government College (A), Anantapur
Andhra Pradesh, 515001, India
e-mail: ravidevi19@gmail.com

A. Venkata Lakshmi
Department of Mathematics
Osmania University
Telangana State, 500007, India
e-mail: akavaramvlr@gmail.com