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ON REFINED YOUNG INEQUALITIES AND REVERSE INEQUALITIES

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Abstract. In this paper, we show refined Young inequalities for two positive operators. Our results refine the ordering relations among the arithmetic mean, the geometric mean and the harmonic mean for two positive operators. In addition, we give two different reverse inequalities for the refined Young inequality for two positive operators.

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