

THE STEIN–WEISS TYPE INEQUALITIES FOR THE B -RIESZ POTENTIALS

A. D. GADJIEV, V. S. GULIYEV, A. SERBETCI AND E. V. GULIYEV

Abstract. We establish two inequalities of Stein-Weiss type for the Riesz potential operator $I_{\alpha,\gamma}$ (B -Riesz potential operator) generated by the Laplace-Bessel differential operator Δ_B in the weighted Lebesgue spaces $L_{p,|x|^\beta,\gamma}$. We obtain necessary and sufficient conditions on the parameters for the boundedness of $I_{\alpha,\gamma}$ from the spaces $L_{p,|x|^\beta,\gamma}$ to $L_{q,|x|^{-\lambda},\gamma}$, and from the spaces $L_{1,|x|^\beta,\gamma}$ to the weak spaces $WL_{q,|x|^{-\lambda},\gamma}$. In the limiting case $p = Q/\alpha$ we prove that the modified B -Riesz potential operator $\tilde{I}_{\alpha,\gamma}$ is bounded from the spaces $L_{p,|x|^\beta,\gamma}$ to the weighted B - BMO spaces $BMO_{|x|^{-\lambda},\gamma}$.

As applications, we get the boundedness of $I_{\alpha,\gamma}$ from the weighted B -Besov spaces $B_{p\theta,|x|^\beta,\gamma}^s$ to the spaces $B_{q\theta,|x|^{-\lambda},\gamma}^s$. Furthermore, we prove two Sobolev embedding theorems on weighted Lebesgue $L_{p,|x|^\beta,\gamma}$ and weighted B -Besov spaces $B_{p\theta,|x|^\beta,\gamma}^s$ by using the fundamental solution of the B -elliptic equation $\Delta_B^{\alpha/2}$.

Mathematics subject classification (2010): 42B20, 42B25, 42B35.

Keywords and phrases: Laplace-Bessel differential operator, B -Riesz potential, Stein-Weiss type inequalities, weighted Lebesgue space, weighted B -Besov space.

REFERENCES

- [1] I.A. ALIEV, B. RUBIN, *Wavelet-like transforms for admissible semi-groups; inversion formulas for potentials and Radon transforms*, J. Fourier Anal. Appl., **11**, 3 (2005), 333–352.
- [2] G. ALTENBURG, *Eine Realisierung der Theorie der abstrakten Besov-Raume $B_q^s(A)$ $s > 0$, $(1 \leq q \leq \infty)$ und der Lebesgue-Raume $H_{p,\gamma}^s$ auf der Grundlage Besselscher Differential-operatoren*, Z. Anal. Anwendungen, **3**, 1 (1984), 43–63.
- [3] M. ASSAL, H. BEN ABDALLAH, *Generalized weighted Besov spaces on the Bessel hypergroup*, J. Funct. Spaces Appl., **4**, 1 (2006), 91–111.
- [4] R.R. COIFMAN, G. WEISS, *Analyse harmonique non commutative sur certains espaces homogènes*, Lecture Notes in Math., **242**, Springer-Verlag, Berlin, 1971.
- [5] D.I. CRUZ-BAEZ AND J. RODRIGUEZ, *Spaces of distributions of Besov and Lizorkin-Triebel type for the Fourier-Bessel transform*, J. Math. Anal. Appl., **259** (2001), 51–63.
- [6] D.E. EDMUNDS, V.M. KOKILASHVILI AND A. MESKHI, *Boundedness and compactness of Hardy-type operators on Banach function spaces defined on measure space*, Proc. A. Razmadze Math. Inst., **117** (1998), 7–30.
- [7] D.E. EDMUNDS, V.M. KOKILASHVILI AND A. MESKHI, *Bounded and Compact Integral Operators*, Dordrecht: Kluwer 2002.
- [8] A.D. GADJIEV, I.A. ALIEV, *On classes of operators of potential types, generated by a generalized shift*, Reports of enlarged Session of the Seminars of I.N. Vekua Inst. of Applied Mathematics, Tbilisi. **3**, 2 (1988), 21–24. (Russian)
- [9] A.D. GADJIEV AND V.S. GULIYEV, *The Stein-Weiss type inequality for fractional integrals, associated with the Laplace-Bessel differential operator*, Fract. Calc. Appl. Anal., **11**, 1 (2008), 77–90.
- [10] E.V. GULIYEV, *On limiting case of the Stein-Weiss type inequality for the B -Riesz potentials*, Fract. Calc. Appl. Anal., **12**, 1 (2009), 1–10.

- [11] E.V. GULIYEV, *Weighted inequality for fractional maximal functions and fractional integrals, associated with the Laplace-Bessel differential operator*, Trans. Natl. Acad. Sci. Azerb. Ser. Phys.-Tech. Math. Sci., **26**, 1 (2006), 71–80.
- [12] V.S. GULIYEV, *Sobolev's theorem for Riesz B-potentials*, Dokl. Akad. Nauk, **358**, 4 (1998), 450–451 (in Russian).
- [13] V.S. GULIYEV, *On maximal function and fractional integral, associated the Bessel differential operator*, Math. Inequal. Appl., **6**, 2 (2003), 317–330.
- [14] V. S. GULIYEV, A. SERBETCI AND I. EKINCIOLGU, *Necessary and sufficient conditions for the boundedness of rough B-fractional integral operators in the Lorentz spaces*, J. Math. Anal. Appl., **336**, 1 (2007), 425–437.
- [15] V.S. GULIEV, A. SERBETCI, AND I. EKINCIOLGU, *On boundedness of the generalized B-potential integral operators in the Lorentz spaces*, Integral Transforms Spec. Funct., **18**, 12 (2007), 885–895.
- [16] V.S. GULIYEV, A. SERBETCI, Z.V. SAFAROV, *Nikolskii-Besov spaces associated with the Fourier-Bessel transforms*, Transactions of NAS of Azerbaijan. Embedding theorems, Harmonic analysis, **13** (2007), 194–211.
- [17] I.A. KIPRIYANOV, *Fourier-Bessel transformations and imbedding theorems*, Trudy Math. Inst. Steklov, **89** (1967), 130–213.
- [18] I.A. KIPRIYANOV, V.I. KONONENKO, *The fundamental solutions for B-elliptic equations*, Differentsialnye Uravneniya, **3**, 1 (1967), 114–129.
- [19] I.A. KIPRIYANOV, L.A. IVANOV, *Obtaining fundamental solutions for homogeneous equations with singularities in several variables*, Trudy Sem. Soboleva, Inst. Mat., Novosibirsk, **1** (1983), 55–77.
- [20] V.M. KOKILASHVILI, A. KUFNER, *Fractional integrals on spaces of homogeneous type*, Comment. Math. Univ. Carolinae, **30**, 3 (1989), 511–523.
- [21] V. KOKILASHVILI, A. MESKHI, *On some weighted inequalities for fractional integrals on nonhomogeneous spaces*, Zeitschrift für Analysis und ihre Anwendungen, **24**, 4 (2005), 871–885.
- [22] B.M. LEVITAN, *Bessel function expansions in series and Fourier integrals*, Uspekhi Mat. Nauk **6**, 2 (42) (1951), 102–143. (Russian)
- [23] L.N. LYAKHOV, *Multipliers of the Mixed Fourier-Bessel Transformation*, Proc. V.A. Steklov Inst. Math., **214** (1997), 234–249.
- [24] R.A. MACIAS, C. SEGOVIA, *A Well Behaved Quasi-distance for Spaces of Homogeneous Type*, Trabajos de Matematica, Instituto Argentino de Matematica, Buenos Aires, **32** (1981), 1–17.
- [25] B. MUCKENHOUPT, *Weighted norm inequalities for the Hardy maximal function*, Trans. Amer. Math. Soc., **165** (1972), 207–226.
- [26] B. MUCKENHOUPT AND E.M. STEIN, *Classical expansions and their relation to conjugate harmonic functions*, Trans. Amer. Math. Soc., **118** (1965), 17–92.
- [27] E.M. STEIN AND G. WEISS, *Fractional integrals on n-dimensional Euclidean spaces*, J. Math. Mech., **7**, 4 (1958), 503–514.
- [28] K. STEMPAK, *The Littlewood-Paley theory for the Fourier-Bessel transform*, Mathematical Institute of Wroclaw (Poland), Preprint no. **45** (1985).
- [29] K. STEMPAK, *Almost everywhere summability of Laguerre series*, Studia Math., **100**, 2 (1991), 129–147.
- [30] K. TRIMÈCHE, *Generalized Harmonic Analysis and Wavelet Packets*, Gordon and Breach Science Publishers, Australia, 2001.