

## QUADRATIC INTERPOLATION AND SOME OPERATOR INEQUALITIES

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**Abstract.** We investigate some properties of Hilbert spaces and bounded linear operators under quadratic interpolation in both qualitative and quantitative ways. Interpolation type, reiteration, interpolation methods associated with quasi-power function parameters, nonlinear commutator estimates, and interpolation of certain operators and spectral properties are under consideration.

**Mathematics subject classification (2010):** Primary 46B70, 46M35; Secondary 46C05, 47A10, 47A63.

**Keywords and phrases:** Interpolation type, quadratic interpolation, quasi-power parameter, reiteration, spectrum.

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