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INEQUALITIES FOR THE SCHWAB–BORCHARDT MEAN AND THEIR APPLICATIONS

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Abstract. Inequalities for the Schwab-Borchardt mean are obtained. They contain known results for the trigonometric and hyperbolic functions including those obtained by J. Wilker [15] and C. Huygens [5]. The main results of this paper can also be utilized to obtain new inequalities for some bivariate means including the logarithmic mean and two means introduced by Seiffert.

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