

MORE ABOUT JENSEN'S INEQUALITY AND CAUCHY'S MEANS FOR SUPERQUADRATIC FUNCTIONS

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Abstract. One of the many inequalities that superquadratic functions satisfy is the parallelogram inequality

$$f(u+v) + f(u-v) \geq 2f(u) + 2f(v).$$

In this paper, we present Cauchy means for superquadratic functions and other mean value theorems. We show also positive semi-definiteness, log-convexity, exponential convexity of certain set of functions.

Mathematics subject classification (2010): Primary 26A51; Secondary 26A46, 26A48.

Keywords and phrases: Jensen's inequality, log-convex function, linear functional, superquadracity, mean value theorems.

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