

## DIRECT AND INVERSE STRONG-TYPE INEQUALITIES FOR JACKSON–MATSUOKA POLYNOMIALS ON THE SPHERE

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**Abstract.** The direct and inverse strong-type inequalities are established for the best approximation by Jackson–Matsuoka polynomials in the  $L_p$  space on the unit sphere of  $\mathbb{R}^d$  which with the help of the relation between K-functionals and modulus of smoothness of sphere.

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