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REVERSES OF ANDO AND DAVIS–CHOI INEQUALITY

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Abstract. Reverse Ando's inequalities for positive linear maps are deduced. A difference reverse inequality is proven for solidarities one of which is of a special type that includes connections. A quotient reverse inequality (or an operator reverse Hölder inequality) for connections and for the special type of solidarities that includes connections is given. As an application an operator reverse of Hölder's inequality for the general weighted power mean is given in a difference and a quotient form. An analogous inequality in a difference form for the relative operator entropy is directly proven. An another type of reverse inequalities is given for Ando's and Davis-Choi's inequality. The estimations in this case are expressed using a kind of variation of involved family of operators.

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