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NECESSARY AND SUFFICIENT CONDITIONS FOR THE VALIDITY OF HILBERT TYPE INTEGRAL INEQUALITIES WITH A CLASS OF QUASI–HOMOGENEOUS KERNELS AND ITS APPLICATION IN OPERATOR THEORY

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Abstract. By using real analysis technique and the method of weight functions, the necessary and sufficient conditions for the validity of Hilbert type integral inequalities with a class of quasi-homogeneous kernels and the best constant factors are obtained, and its applications in operator theory are discussed.

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