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INTEGRAL OPERATORS WITH TWO VARIABLE INTEGRATION LIMITS ON THE CONE OF MONOTONE FUNCTIONS

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Abstract. Weighted inequalities for the Hardy-Steklov operators with variable integration limits on the cone of monotone functions have been investigated with success. The similar problem for operators with kernels has remained unsolved up to now. In this paper we find characterizations for integral operators with a wide class of kernels.

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