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LAWS OF LARGE NUMBERS WITH INFINITE MEAN

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Abstract. In this paper, we study the weak law and strong law of large numbers based on $\tilde{\rho}$-mixing random variables with infinite mean. If the random variables satisfy the Pareto type distributions, then some weak laws of large numbers are presented. If the random variables satisfy the two tailed Pareto distribution and asymmetrical Cauchy distribution, the strong laws of large numbers are also obtained. Furthermore, we do some simulations for the laws of large numbers for two tailed Pareto distribution and asymmetrical Cauchy distribution.

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REFERENCES

[1] ADLER, A., ROSALSKY, A., *Some general strong laws for weighted sums of stochastically dominated random variables*, Stoch. Anal. Appl. **5**(1987), 1–16.
[2] ADLER, A., ROSALSKY, A., TAYLOR, R.L., *Strong laws of large numbers for weighted sums of random elements in normed linear spaces*, Int. J. Math. Math. Sci. **12**(1989), 507–530.
[3] ADLER, A., *On the nonexistence of a law of the iterated logarithm for weighted sums of identically distributed random variables*, J. Appl. Math. Stoch. Anal. **3**(1990), 135–140.
[4] ADLER, A., *Generalized one-sided laws of the iterated logarithm for random variables barely with or without finite mean*, J. Theoret. Probab. **3**(1990), 587–597.
[5] ADLER, A., *Laws of large numbers for asymmetrical Cauchy random variables*, J. Appl. Math. Stoch. Anal. (2007), 1-6 Article ID 56924.
[6] ADLER, A., *Laws of large numbers for two tailed Pareto random variables*, Probab. Math. Statist. **28**(2008), 121–128.
[7] ADLER, A., *An exact weak law of large numbers*, Bull. Inst. Math. Acad. Sin. (NS) **7**(2012), 417–422.
[8] AN, J., YUAN, D.M., *Complete convergence of weighted sums for ρ^*-mixing sequence of random variables*, Stat. Probab. Lett. **78**(2008), 1466–1472.
[9] BRADLEY, R.C., *On the spectral density and asymptotic normality of weakly dependent random fields*, J. Theoret. Probab. **5**(1992), 355–373.
[10] BRADLEY, R.C., *Every "lower psi-mixing" Markov chain is "interlaced rho-mixing"*, Stochastic Process. Appl. **72**(1997), 221–239.
[11] BRADLEY, R.C., *Basic Properties of Strong Mixing Conditions. A Survey and Some Open Questions*, Probab. Surveys **2**(2005), 107–144.
[12] CHOW, Y.S., TEICHER, H., *Probability Theory: Independence, Interchangeability, Martingales. 3rd ed.*, Springer-Verlag, New York (1997).
[13] GAN, S.X., *Almost sure convergence for $\tilde{\rho}$-mixing random variable sequences*, Stat. Probab. Lett. **67**(2004), 289–298.
[14] GUT, A., *Probability: A Graduate Course. 2rd ed.*, Springer Science+Business Media, New York (2013).
[15] KUCZMASZEWSKA, A., *On complete convergence for arrays of rowwise dependent random variables*, Stat. Probab. Lett. **77**(2007), 1050–1060.
[16] LI, W., CHEN, P.Y., SUNG, S.H., *Remark on convergence rate for weighted sums of ρ^*-mixing random variables*, RACSAM, **111**(2017), 507–513.

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[17] MATSUMOTO, K., NAKATA, T., *Limit theorems for a generalized Feller game*, J. Appl. Probab. **50**(2013): 54–63.

[18] NAKATA, T., *Limit theorems for nonnegative independent random variables with truncation*, Acta Math. Hungar. **145**(2015): 1–16.

[19] NAKATA, T., *Weak laws of large numbers for weighted independent random variables with infinite mean*, Stat. Probab. Lett. **109**(2016), 124–129.

[20] STEIN, S., *A bound for the error in the normal approximation to the distribution of a sum of dependent random variables*, Proceedings of the Sixth Berkeley Symposium on Mathematical Statistics and Probability Vol 2, University of California Press, Berkeley, CA, 1972, 583–602.

[21] SUNG, S.H., *On the strong convergence for weighted sums of ρ^*-mixing random variables*, Stat. Pap. **54**(2013), 773–781.

[22] UTEV, V., PELIGRAD, M., *Maximal inequalities and an invariance principle for a class of weakly dependent random variables*, J. Theoret. Probab. **16**(2003), 101–115.

[23] WANG, X.J., DENG, X., XIA, F.X., HU, S.H., *The consistency for the estimators of semiparametric regression model based on weakly dependent errors*, Stat. Pap. **58**(2017), 303–318.

[24] WANG, X.J., LI, X.Q., YANG, W.Z., HU, S.H., *On complete convergence for arrays of rowwise weakly dependent random variables*, Appl. Math. Lett. **25**(2012), 1916–1920.

[25] WANG, Y.W., YANG, W.Z., HU, S.H., *The Bahadur representation of sample quantiles for weakly dependent sequences*, Stoch.: Int. J. Probab. Stoch. Process, **88**(2016), 428–436.

[26] WU, Q.Y., JIANG, Y.Y., *Some strong limit theorems for $\tilde{\rho}$-mixing sequences of random variables*, Stat. Probab. Lett. **78**(2008), 1017–1023.