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# A NONLINEAR VERSION OF HALANAY INEQUALITY AND APPLICATION TO NEURAL NETWORKS THEORY

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*Abstract.* We establish an exponential stability result for a delayed Hopfield neural network. This is proved in case one or more of the activation functions fails to satisfy the standard Lipschitz continuity condition. We use a nonlinear version of Halanay inequality, which we prove here.

*Mathematics subject classification* (2010): 92B20, 93D20, 34D20, 37C75.

*Keywords and phrases*: Hopfield neural network, exponential stabilization, non-Lipschitz continuous activation functions, nonlinear Halanay inequality.

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