Journal of
Mathematical
Inequalities
Volume 15, Number 1 (2021), 173–200 doi:10.7153/jmi-2021-15-15

SOME GENERALIZED GRONWALL–BELLMAN TYPE DIFFERENCE INEQUALITIES AND APPLICATIONS

ZIZUN LI

Abstract. We establish some generalized sums-difference inequalities involving a finite sum, which includes three sums and seven sums, respectively. We present the estimation of the inequality is decided by a finite recursion. Using the lemma and difference techniques, we transform the complex difference inequalities into the simple forms of difference inequalities. We apply our results to boundary value problem of a partial difference equation for uniform boundedness, uniqueness and continuous dependence of the solutions.

Mathematics subject classification (2010): 34B15, 26D15, 26D10.
Keywords and phrases: Difference inequality, monotonicity, boundary value problem, asymptoticity, boundedness.

REFERENCES

[1] A. ABDELDAIM AND A. A. EL-DEEB, *On generalized of certain retarded nonlinear integral inequalities and its applications in retarded integrodifferential equations*, Applied Mathematics and Computation, 256, 375–380 (2015).
[2] A. ABDELDAIM, A. A. EL-DEEB, AND REDA GAMAL AHMED, *On retarded nonlinear integral inequalities of Gronwall and applications*, Journal of Mathematical Inequalities, Volume 13, Number 4 (2019), 1023–1038.
[3] R. P. AGARWAL, *Difference equations and inequalities*, Marcel Dekker, New York, 1992.
[4] P. Y. H. PANG, R. P. AGARWAL, *On an integral inequality and its discrete analogue*, J. Math. Anal. Appl. 194 (1995) 569–577.
[5] R. P. AGARWAL, S. F. DENG, W. N. ZHANG, *Generalization of a retarded Gronwall-like inequality and its applications*, Appl. Math. Comput. 165 (2005) 599–612.
[6] R. P. AGARWAL, Y. H. KIM, S. K. SEN, *New retarded integral inequalities with applications*, J. Inequ. Appl. 2008 (2008) 15 pages.
[7] R. BELLMAN, *The stability of solutions of linear differential equations*, Duke Math. J. 10 (1943) 643–647.
[8] D. BAINOV, P. SIMEONOV, *Integral Inequalities and Applications*, Kluwer Academic, Dordrecht, 1992.
[9] I. A. BIHARI, *A generalization of a lemma of Bellman and its application to uniqueness problem of differential equation*, Acta Math. Acad. Sci. Hung. 7 (1956) 81–94.
[10] C. J. CHEN, W. S. CHEUNG AND D. ZHAO, *Gronwall-Bellman-Type integral inequalities and applications to BVPs*, J. Inequ. Appl. 2009 (2009) Art. ID 258569 15 pages.
[11] W. S. CHEUNG, *Some discrete nonlinear inequalities and applications to bounadry value problems for difference equations*, J. Differ. Equat. Appl. 10 (2004) 213–223.
[12] W. S. CHEUNG, *Some new nonlinear inequalities and applications to boundary value problems*, Nonlinear Anal. 64 (2006) 2112–2128.
[13] W. S. CHEUNG, *Some retarded Gronwall-Bellman-Ou-Iang-type inequalities and applications to initial boundary value problems*, preprint.
[14] W. S. CHEUNG, J. REN, *Discrete non-linear inequalities and applications to boundary value problems*, J. Math. Anal. Appl. 319 (2006) 708–724.

© ELEMENT, Zagreb
Paper JMI-15-15

[15] W. S. CHEUNG, Q. H. MA, J. PECARIC, *Some discrete nonlinear inequalities and applications to difference equations*, Acta Math. Sci. 28 (2) (2008) 417–430.

[16] S. K. CHOI, S. F. DENG, N. J. KOO AND W. N. ZHANG, *Nonlinear Integral Inequalities of Bihari-Type without Class H*, Math. Inequ. Appl. 8 (2005) 643–654.

[17] S. S. DRAGOMIR, Y. H. KIM, *Some integral inequalities for functions of two variables*, Electr. J. Diff. Eqns. 2003 (2003) No. 10 1–13.

[18] A. A. EL-DEEB, *On some generalizations of nonlinear dynamic inequalities on time scales and their applications*, Applicable Analysis and Discrete Mathematics. 13 (2019), 440–462.

[19] A. A. EL-DEEB, HAIYONG XU, A. ABDELDAIM, AND GUOTAO WANG, *Some dynamic inequalities on time scales and their applications*, Advances in Difference Equations, (2019) 130 (2019), `https://doi.org/10.1186/s13662-019-2023-6`.

[20] Q. H. FENG, F. W. MENG, B.S. FU, *Some new generalized Volterra-Fredholm type finite difference inequalities involving four iterated sums*, Appl. Math. Comput. 219 (2013) 8247–8258.

[21] T. H. GRONWALL, *Note on the derivatives with respect to a parameter of the solutions of a system of differential equations*, Ann. of Math. 20 (1919) 292–296.

[22] Y. H. KIM, *Gronwall, Bellman and Pachpatte type integral inequalities with applications*, Nonlinear Anal. 71 (2009) 2641–2656.

[23] O. LIPOVAN, *Integral inequalities for retarded Volterra equations*, J. Math. Anal. Appl. 322 (2006) 349–358.

[24] Z. Z. LI, *A generalized nonlinear sums-difference inequality and its applications*, J. Math. Inequ. 12 (2018) 77–94.

[25] T. LV, Y. HUANG, *A generalization of discrete Gronwall inequality and its application to weakly singular Volterra integral equation of the second kind*, J. Math. Anal. Appl. 282 (2003) 56–62.

[26] Q. H. MA, E. H. YANG, *Some new Gronwall-Bellman-Bihari type integral inequalities with delay*, Periodica Mathematica Hungarica. 44 (2002) 225–238.

[27] Q. H. MA, W. S. CHEUNG, *Some new nonlinear difference inequalities and their applications*, J. Comput. Appl. Math. 202 (2007) 339–351.

[28] Q. H. MA, *Some new nonlinear Volterre-Fredholm type discrete inequalities and their applications*, J. Comput. Appl. Math. 216 (2008) 451–466.

[29] Q. H. MA, *Estimates on some power nonlinear Volterra–Fredholm type discrete inequalities and their applications*, J. Comput. Appl. Math. 233 (2010) 2170–2180.

[30] F. W. MENG, W. N. LI, *On some new nonlinear discrete inequalities and their applications*, J. Comput. Appl. Math. 158 (2003) 407–417.

[31] F. W. MENG, D. H. JI, *On some new nonlinear discrete inequalities and their applications*, J. Comput. Appl. Math. 208 (2007) 425–433.

[32] B. G. PACHPATTE, *On some new inequalities related to certain inequalities in the theory of differential equations*, J. Math. Anal. Appl. 189 (1995) 128–144.

[33] B. G. PACHPATTE, *Inequalities for Differential and Integral Equations*, Academic Press, New York, 1998.

[34] B. G. PACHPATTE, *On some new inequalities related to a certain inequality arising in the theory of differential equations*, J. Math. Anal. Appl. 251 (2000) 736–751.

[35] B. G. PACHPATTE, *Integral and finite difference inequalities and applications*, vol. 205 of North-Holland Mathematics Studies, Elsevier Science B. V., Amsterdam, the Netherlands, 2006.

[36] H. Y. QIN, X. ZUO, J. W. LIU, *Some New Generalized Retarded Gronwall-Like Inequalities and Their Applications in Nonlinear Systems*, J. Contr. Sci. Engr. 2016 (2016) Art. ID 9527680, 8 pages.

[37] SH. SALEM, *Note on some systems of two discrete inequalities of Gronwall type*, J. Math. Anal. Appl. 208 (1997) 553–566.

[38] W. H. SHENG, W. N. LI, *Bounds on certain nonlinear discrete inequalities*, J. Math. Inequ. 2 (2) (2008) 279–286.

[39] YAZHOU TIAN, A. A. EL-DEEB, AND FANWEI MENG, *Some nonlinear delay Volterra–Fredholm type dynamic integral inequalities on time scales*, Discrete Dynamics in Nature and Society, Volume 2018, Article ID 5841985, 8 pages.

[40] W. S. WANG, *A generalized sum-difference inequality and applications to partial Difference equations*, Adv. Difference Equ. 2008 (2008) 12 pages.

[41] W. S. WANG, X. L. ZHOU, *An extension to nonlinear sum-difference inequality and applications*, Adv. Difference Equ. 2009 (2009) 17 pages.

[42] W. S. WANG, Z. Z. LI, Y. LI, Y. HUANG, *Nonlinear retarded integral inequalities with two variables and applications*, J. Inequ. Appl. 2010 (2010) 21 pages.
[43] W. S. WANG, Z. Z. LI , W. S. CHEUNG, *Some new nonlinear retarded sum-difference inequalities with applications*, Adv. Difference Equ. 2011 (2011) 11 pages.
[44] D. WILLETT, J. S. W. WONG, *On the discrete analogues of some generalizations of Gronwall's inequality*, Monatsh. Math. 69 (1965) 362–367.
[45] Y. WU, X. P. LI, S. F. DENG, *Nonlinear delay discrete inequalities and their applications to Volterra type difference equations*, Adv. Difference Equ. 2010 (2010) 14 page.
[46] E. H. YANG, *On some new discrete Generalizations of Gronwall's inequality*, J. Math. Anal. Appl. 129 (1988) 505–516.
[47] W. ZHANG, S. DENG, *Projected Gronwall-Bellman's inequality for integrable functions*, Math. Comput. Modelling. 34 (2001) 394–402.
[48] B. ZHENG, B. FU, *Some Volterra-Fredholm type nonlinear discrete inequalities involving four iterated infinite sums*, Adv. Difference. Equ. 2012 (2012) 18 pages.
[49] K. L. ZHENG, Y. WU, S. M. ZHONG, *Discrete nonlinear intrgral inequalities in two variables and their applications*, Appl. Math. Comput. 207 (2009) 140–147.
[50] L. F. ZHOU, W. N. ZHANG, *A projected discrete Gronwall's inequality with sub-exponential growth*, J. Difference Equ. Appl. 16 (8) 2010 931–943.
[51] J. ZHOU, J. SHEN, W. N. ZHANG, *A powered Gronwall-type inequality and applications to stochastic differential equations*, Discr. Conti. Dyna. Syst. 36 (12) (2016) 7207–7234.

Journal of Mathematical Inequalities
www.ele-math.com
jmi@ele-math.com