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A NEW GENERALIZED REFINEMENTS OF YOUNG'S INEQUALITY AND APPLICATIONS

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Abstract. In this work, by the weighted arithmetic-geometric mean inequality, we show if $a,b>0$ and $0\leqslant \nu\leqslant 1$. Then for all positive integer m, we have

$$\begin{aligned}\left(a^{\nu}b^{1-\nu}\right)^{m} &+ r_0^m\left((a+b)^m-2^m(ab)^{\frac{m}{2}}\right)\\ &+r_m\left[\left((ab)^{\frac{m}{4}}-b^{\frac{m}{2}}\right)^2\chi_{(0,\frac{1}{2}]}(\nu)+\left((ab)^{\frac{m}{4}}-a^{\frac{m}{2}}\right)^2\chi_{(\frac{1}{2},1]}(\nu)\right]\\ &\leqslant \left(\nu a+(1-\nu)b\right)^m,\end{aligned}$$

where $r_0=\min\{\nu,1-\nu\}$, $r_m=\min\{2^m r_0^m,(1-r_0)^m-r_0^m\}$ and $\chi_I(\nu)$ the characteristic function. This inequality provides a generalization of an important refinement of the Young inequality obtained by J. Zhao and J. Wu. As applications we give some new generalized refinements of Young type inequalities for the determinants, p-norms and traces, of positive τ-measurable operators.

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