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MAHGOUB TRANSFORM AND HYERS–ULAM STABILITY OF FIRST–ORDER LINEAR DIFFERENTIAL EQUATIONS

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Abstract. The main aim of this paper is to investigate various types of Hyers-Ulam stability of linear differential equations of first order with constant coefficients using the Mahgoub transform method. We also show the Hyers-Ulam constants of these differential equations and give some examples to better illustrate the main results.

Mathematics subject classification (2020): 34A40, 39B82, 26D10, 44A15.
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