

## INEQUALITIES AND NUMERICAL RESULTS OF APPROXIMATION FOR BIVARIATE $q$ -BASKAKOV-DURRMEYER TYPE OPERATORS INCLUDING $q$ -IMPROPER INTEGRAL

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**Abstract.** In this study, we investigate inequalities estimating the error of the approximation of bivariate  $q$ -Baskakov-Durrmeyer type operators including  $q$ -improper integral. We firstly introduce bivariate  $q$ -Baskakov-Durrmeyer type operators including the  $q$ -improper integral. We obtain inequalities estimating the error of the approximation for these operators. Later, we introduce generalized Boolean sum (GBS) operators associated to the bivariate  $q$ -Baskakov-Durrmeyer type operators including the  $q$ -improper integral, and we give an inequality estimating the error of the approximation for the GBS operators. Lastly, we present numerical results of error estimations for certain functions with the help of maple software.

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