

## ON THE $q$ -ANALOGUES FOR SOME KANTOROVICH TYPE LINEAR OPERATORS

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**Abstract.** In this paper, we present some Kantorovich type positive linear operators and we introduce the new modification  $q$ -Baskakov-Kantorovich operators. We prove the convergence of the new operators using the Korovkin criterion and establish the rate of convergence involving the modulus of continuity. Also, some numerical results to illustrate the convergence of these operators.

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