

## SOME CHARACTERIZATIONS OF $h$ -CONVEX FUNCTIONS

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*Abstract.* In this paper, we give some characterizations of  $h$ -convex functions, and some applications related to these functions are also obtained. According to these results, we can know better the relation between convex functions and  $h$ -convex functions.

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## REFERENCES

- [1] M. ALOMARI, *A note on  $h$ -convex functions*, e-J. Anal. Appl. Math., **2019** (1) (2019), 55–67.
- [2] P. BURAI AND A. HÁZY, *On approximately  $h$ -convex functions*, J. Convex Anal., **18** (2) (2011), 1–9.
- [3] M. BOMBARELLI AND S. VAROŠANEC, *Properties of  $h$ -convex functions related to the Hermite-Hadamard-Fejér inequalities*, Comput. Math. Appl., **58** (2009), 1869–1877.
- [4] W. BRECKNER, *Stetigkeitsaussagen für eine Klasse verallgemeinerter Konvexer funktionen in topologischen linearen Räumen*, Publ. Inst. Math., **23** (1978), 13–20.
- [5] M. DELAVAR, S. DRAGOMIR AND M. DE LA SEN, *A note on characterization of  $h$ -convex functions via Hermite-Hadamard type inequality*, Probl. Anal. Issues Anal., **8** (26), (2019), 28–36.
- [6] S. DRAGOMIR, *Inequalities of Jensen type for  $h$ -convex functions on linear spaces*, Math. Moravica, **19** (1) (2015), 107–121.
- [7] S. DRAGOMIR, *Inequalities of Hermite-Hadamard type for  $h$ -convex functions on linear spaces*, Proyecciones J. Math., **34** (4) (2015), 323–341.
- [8] S. DRAGOMIR, J. PEČARIĆ AND L. PERSSON, *Some inequalities of Hadamard type*, Soochow J. Math., **21** (1995), 335–341.
- [9] M. FENG, J. RUAN AND X. MA, *Hermite-Hadamard inequalities for multidimensional strongly  $h$ -convex functions*, Math. Inequal. Appl., **24** (4) (2021), 897–911.
- [10] E. GODUNOVA AND V. LEVIN, *Neravenstva dlja funkciij širokogo klassa, soderžaščego vypuklye, monotonne i nekotorye drugie vidy funkciij*, Vyčislitel. Mat. i. Mat. Fiz. Mežvuzov. Sb. Nauč. Trudov, MGPI, Moskva, (1985), 138–142.
- [11] A. HÁZY, *Bernstein-Doetsch type results for  $h$ -convex functions*, Math. Inequal. Appl., **14** (3) (2011), 499–508.
- [12] M. LATIF, *On some inequalities for  $h$ -convex functions*, Int. J. Math. Anal., **4** (30) (2010), 1473–1482.
- [13] M. MATŁOKA, *On Hadamard's inequality for  $h$ -convex function on a disk*, Appl. Math. Comput., **235** (2014), 118–123.
- [14] D. MITRINović AND J. PEČARIĆ, *Note on a class of functions of Godunova and Levin*, C. R. Math. Rep. Acad. Sci. Can., **12** (1990), 33–36.
- [15] D. MITRINović, J. PEČARIĆ AND A. FINK, *Classical and new inequalities in analysis*, Kluwer Academic, Dordrecht, 1993.
- [16] A. OLBRYŚ, *Representation theorems for  $h$ -convexity*, J. Math. Anal. Appl., **426** (2015), 986–994.
- [17] M. ÖZDEMİR, M. GÜRBÜZ, AND A. AKDEMİR, *Inequalities for  $h$ -convex functions via further properties*, RGMIA Research Report Collection, **14**, (22) (2011).
- [18] M. ÖZDEMİR, M. TUNC AND M. GÜRBÜZ, *Definitions of  $h$ -logarithmic,  $h$ -geometric and  $h$ -multi convex functions and some inequalities related to them*, arXiv:1211.2570v1, (2012), 1–8.
- [19] C. PEARCE AND A. RUBINOV,  *$P$ -functions, quasi-convex functions and Hadamard-type inequalities*, J. Math. Anal. Appl., **240** (1999), 92–104.

- [20] M. SARIKAYA, A. SAGLAM AND H. YILDRIM, *On some Hadamard-type inequalities for  $h$ -convex functions*, J. Math. Inequal., **2** (3) (2008), 335–341.
- [21] M. TUNC, *On some integral inequalities via  $h$ -convexity*, Miskolc Math. Notes, **14** (3) (2013), 1041–1057.
- [22] M. TUNC, *Ostrowski-type inequalities via  $h$ -convex functions with applications to special means*, J. Inequal. Appl., **326** (2013), 1–10.
- [23] M. TUNC AND H. YILDRIM, *New estimations for  $h$ -convex functions via further properties*, arXiv:1202.2207v1, (2012), 1–11.
- [24] M. TUNC, H. YILDRIM AND A. EKİNÇİ, *On some inequalities of Simpson's type via  $h$ -convex functions*, Hacettepe J. Math. Stat., **42** (4) (2013), 309–317.
- [25] S. VAROŠANEC, *On  $h$ -convexity*, J. Math. Anal. Appl., **326** (2007), 303–311.
- [26] T. WANG, M. FENG, J. RUAN AND B. SHAO, *Hermite-Hadamard inequalities for co-ordinated log- $h$ -convex functions*, J. Math. Inequal., **15** (1) (2021), 31–46.
- [27] X. WANG, J. RUAN AND X. MA, *On the Hermite-Hadamard inequalities for  $h$ -convex functions on balls and ellipsoids*, Filomat, **33** (18) (2019), 5817–5886.
- [28] D. ZHAO, T. AN, G. YE AND W. LIU, *New Jensen and Hermite–Hadamard type inequalities for  $h$ -convex interval-valued functions*, J. Inequal. Appl., **302** (2018), 1–14.