

ALTERNATIVE PROOFS OF SHAFER'S INEQUALITY FOR INVERSE HYPERBOLIC TANGENT

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Abstract. We point out that a concise proof of Theorem 2 in the article, '*On a quadratic estimate of Shafer*' by L. Zhu contains a small mistake. Correcting this mistake and giving alternative proofs of Theorem 2 is the main aim of this note.

Mathematics subject classification (2020): 26A09, 26D05, 26D15.

Keywords and phrases: Shafer's inequality, inverse hyperbolic tangent, increasing-decreasing function.

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[doi:10.7153/jmi-02-51](https://doi.org/10.7153/jmi-02-51).