

## ON MULTIPLICATIVE SUM ZAGREB INDEX OF TREES WITH FIXED DOMINATION NUMBER

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**Abstract.** The multiplicative sum Zagreb index of a graph  $G$  is the product of the sums of degrees of pairs of adjacent vertices. In this paper, the maximal and minimal multiplicative sum Zagreb indices of trees with fixed domination number are presented. Furthermore, the corresponding extremal trees are identified.

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