

## A MAP-TYPE GRONWALL INEQUALITY ON FUNCTIONAL DIFFERENTIAL EQUATIONS WITH STATE-DEPENDENCE

JUN ZHOU

**Abstract.** In this paper, in order to investigate a Gronwall inequality with state-dependence, another auxiliary map-type Gronwall inequality is discussed by modifying the technique of sequential monotization on functions into the one on maps. Then we employ the state-dependent Gronwall inequality to give the estimate and boundedness of solutions for a functional differential equation with state-dependence. Finally, we exhibit a concrete example of bounded solutions as application.

**Mathematics subject classification (2020):** 26D15, 34K25.

**Keywords and phrases:** Gronwall inequality, functional differential equations, state-dependence, estimate, boundedness.

### REFERENCES

- [1] R. P. AGARWAL, S. DENG, W. ZHANG, *Generalization of a retarded Gronwall-like inequality and its applications*, Appl. Math. Comput., **165** (2005), 599–612.
- [2] R. BELLMAN, *The stability of solutions of linear differential equations*, Duke Math. J., **10** (1943), 643–647.
- [3] I. A. BIHARI, *A generalization of a lemma of Bellman and its application to uniqueness problem of differential equation*, Acta Math. Acad. Sci. Hung., **7** (1956), 81–94.
- [4] T. H. GRONWALL, *Note on the derivatives with respect to a parameter of the solutions of a system of differential equations*, Ann. Math., **20** (1919), 292–296.
- [5] J. HÄRTERICH, B. SANDSTEDTE, A. SCHEEL, *Exponential dichotomies for linear non-autonomous functional differential equations of mixed type*, Indiana Univ. Math. J., **51** (2020), 1081–1109.
- [6] O. LIPOVAN, *A retarded Gronwall-like inequality and its applications*, J. Math. Anal. Appl., **252** (2000), 389–401.
- [7] J. MALLET-PARET, *The Fredholm alternative for functional differential equations of mixed type*, J. Dyn. Diff. Eq., **11** (1999), 1–47.
- [8] B. G. PACHPATTE, *Inequalities for Differential and Integral Equations*, Academic Press, New York, 1998.
- [9] M. PINTO, *Integral inequalities of Bihari-type and applications*, Funkcial. Ekvac., **33** (1990), 387–403.
- [10] A. RUSTICHINI, *Hopf bifurcation for functional differential equations of mixed type*, J. Dyn. Diff. Eq., **1** (1989), 145–177.
- [11] W. WANG, *A generalized retarded Gronwall-like inequality in two variables and applications to BVP*, Appl. Math. Comput., **191** (2007), 144–154.
- [12] Y. ZENG, P. ZHANG, T.-T. LU, W. ZHANG, *Existence of solutions for a mixed type differential equation with state-dependence*, J. Math. Anal. Appl., **453** (2017), 629–644.
- [13] J. ZHOU, J. SHEN, W. ZHANG, *A powered Gronwall-type inequality and applications to stochastic differential equations*, Disc. Contin. Dyn. Sys., **36** (2016), 7207–7234.
- [14] J. ZHOU, J. SHEN, W. ZHANG, *Growth in an impulsive integral inequality*, Appl. Math. Comput., **377** (2020), 125152.
- [15] J. ZHOU, *Solutions of mixed-type functional differential equations with state-dependence*, J. Differ. Equ., **312** (2022), 148–175.