

A NEW HILBERT-TYPE INEQUALITY IN THE WHOLE PLANE

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Abstract. By means of the weight coefficients and the idea of introduced parameters, a new discrete Hilbert-type inequality in the whole plane is given, which is an extension of Hardy-Hilbert's inequality. The equivalent form is obtained. The equivalent statements of the best possible constant factor related to several parameters, the operator expressions and a few particular inequalities are considered.

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