

## BOUNDEDNESS FROM BELOW OF COMPOSITION OPERATORS BETWEEN $L_a^p$ AND $L_a^q$ , BETWEEN $L_a^p$ AND THE HARDY SPACE $H^2$ , BETWEEN $L_a^p$ AND BESOV SPACE

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*Abstract.* We study the relation between the composition operators  $C_\phi$  with closed range on the weighted Bloch spaces and  $C_\phi$  with closed range on the weighted Dirichlet spaces  $D_p^\alpha$ . In particular, we study the boundedness from below of composition operators between  $L_a^p$  and  $L_a^q$ , between  $L_a^p$  and Hardy space, and between  $L_a^p$  and Besov space.

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