

ON A MORE ACCURATE REVERSE HARDY–HILBERT’S INEQUALITY WITH TWO PARTIAL SUMS

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Abstract. By means of the weight coefficients, Hermite-Hadamard’s inequality, Euler-Maclaurin summation formula and Abel’s summation by parts formula, a more accurate reverse Hardy-Hilbert’s inequality with two partial sums is given. The equivalent statements of the best possible constant factor related to several parameters are provided, and some particular inequalities are deduced.

Mathematics subject classification (2020): 26D15, 26D10, 47A05.

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