

CHARACTERIZATIONS OF LIPSCHITZ FUNCTIONS VIA THE COMMUTATORS OF FRACTIONAL MAXIMAL FUNCTION IN ORLICZ SPACES ON STRATIFIED LIE GROUPS

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Abstract. In this paper we obtain necessary and sufficient conditions for the boundedness of the fractional maximal commutators $M_{b,\alpha}$ and the commutators of the fractional maximal operator $[b, M_\alpha]$ in Orlicz spaces $L^\Phi(\mathbb{G})$ on any stratified Lie group \mathbb{G} when b belongs to Lipschitz spaces $\dot{\Lambda}_\beta(\mathbb{G})$. We obtain some new characterizations for certain subclasses of Lipschitz spaces $\dot{\Lambda}_\beta(\mathbb{G})$.

Mathematics subject classification (2020): 42B25, 42B35, 43A80, 46E30.

Keywords and phrases: Stratified group, Orlicz space, fractional maximal function, commutator, Lipschitz function.

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