

## FURTHER REFINEMENTS AND GENERALIZATIONS OF THE YOUNG AND ITS REVERSE INEQUALITIES

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**Abstract.** In this work, we mainly refine some celebrated Young-type inequalities. As an application, we present corresponding operators, determinants and matrix inequalities following from the established scalar inequalities.

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