

## THE HERMITE—HADAMARD'S INEQUALITY OVER $p$ -SPHERES AND RELATED INEQUALITIES FOR THE INTEGRAL MEANS

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**Abstract.** The goal of this work is to extend the Hermite-Hadamard inequality, to the  $p$ -sphere. Additionally, we present Trapezoid and Midpoint type inequalities for the integral means of the generalized Hermite-Hadamard inequality. Specifically, for functions defined on  $p$ -spheres that meet certain Hölder-type conditions, we establish corresponding Trapezoid and Midpoint type inequalities.

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