

SHARP RAMANUJAN TYPE INEQUALITIES WITH $1/(x+c)$

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Abstract. We establish the new Ramanujan type inequalities with $\frac{1}{x+c}$ as follows: for $x > 0$, we have

$$\frac{1}{x+\alpha} < \sum_{k=1}^{\infty} \frac{k^{k-2}}{(x+k)^k} < \frac{1}{x+\beta},$$

where the constants $\alpha = \frac{6}{\pi^2} \cong 0.607927$ and $\beta = 0$ are the best possible.

Mathematics subject classification (2020): 26D15, 26A48, 26A60.

Keywords and phrases: Ramanujan's inequality, best possible constant, monotonically decreasing function, monotonically increasing function.

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