

CONSTRUCTION CONDITIONS OF BOUNDED OPERATORS WITH SUPER-HOMOGENEOUS KERNELS IN HIGH-DIMENSIONAL SPACE AND THE ESTIMATION OF OPERATOR NORM

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Abstract. The concept of super-homogeneous functions is introduced, and the construction conditions of bounded operators with super-homogeneous kernels between high-dimensional weighted Lebesgue function spaces and weighted Hilbert-type sequence spaces are discussed. The sufficient and necessary conditions for operator boundedness and the estimation formula for operator norm are obtained, and the exact calculation formula for operator norm is obtained under specific conditions. Finally, several special cases of applications are presented.

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