

## SOME NEW INEQUALITIES FOR (A–A) AND (G–A) $m$ –CONVEX FUNCTIONS WITH APPLICATIONS

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**Abstract.** In this paper, we introduce several new inequalities for (A–A) and (G–A)  $m$ -convex functions. Additionally, we examine multi-term refinements of the classical Jensen-type inequality for these classes of functions. Our results are further extended by applying the weak sub-majorization theory. As an application, we derive new refinements of the classical Hermite-Hadamard inequality for  $m$ -convex functions. Our findings extend and generalize recent publications in this field, including [5, 6, 8, 9, 16].

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