

ON STARLIKENESS OF THE GENERALIZED MARCUM Q -FUNCTION

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Abstract. Our aim in this paper is to present sufficient conditions on the parameters of some classes of analytic functions related to the generalized Marcum Q -function to belong to a certain class of starlike functions. Furthermore, two classes of starlike functions related to the lower incomplete generalized hypergeometric functions are derived. Applications of these are given in the form of corollaries and examples. The key tool in the proofs of the main results are the monotonicity property for the gamma function and an inequality of the lower incomplete gamma function.

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